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New Quantum Effect for Vaidya–Bonner–de Sitter Black Holes

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The field equations of spin $s = 0$, 1/2, 1, and 2 in the Vaidya–Bonner–de Sitter space–time are investigated. The results show that the radiative mechanism of massive spin fields depends on the spin state. We have sufficient reasons to conjecture that the effect originates from the quantum ergosphere.

1. INTRODUCTION

Ever since Hawking (1974) initiated the discussion of particle creation by a black–hole event horizon, much work, especially with the aid of the Newman–Penrose (1962) formalism, has been done on black holes in different types of space–time, such as the Schwarzschild (Page, 1976) and Vaidya (Kim *et al.*, 1989) space–times. The purpose of this paper is to extend this method to the Vaidya–Bonner–de Sitter space–time.

This paper is organized as follows. In Section 2 we calculate the spin coefficients and the tetrad components of the Weyl tensor, and show that the Vaidya–Bonner–de Sitter metric is of Petrov type D. In Section 3 we use the Newman–Penrose formalism, write a master equation for fields of arbitrary spin ($s = 0$ for the scalar field, $s = 1/2$ for the Dirac field, $s = 1$ for the electromagnetic field, $s = 2$ for the gravitational field, etc). In Section 4 we study the character of spin fields near the black-hole event horizon, and show that the quantum ergosphere (York, 1983) can influence the radiative mechanism of a black hole.

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2. VAIDYA–BONNER–de SITTER METRIC

The line element of the space–time is (Patino and Rago, 1987)

$$ds^2 = \left[1 - \frac{2M(v)}{r} + \frac{Q^2(v)}{r^2} - \frac{1}{3}\Lambda r^2\right] dv^2 - 2\,dv\,dr - r^2\,(d\theta^2 + \sin^2\theta\,d\varphi^2) \tag{1}$$

where Λ is the cosmological constant, and both mass M and charge Q depend on the advanced Eddington–Finkelstein time v.

The null tetrad is established as follows:

$$\begin{aligned} l_\mu &= -\delta_\mu^0 \\ n_\mu &= -\frac{1}{2}\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{1}{3}\Lambda r^2\right)\delta_\mu^0 + \delta_\mu^1 \\ m_\mu &= -\frac{r}{\sqrt{2}}\,(\delta_\mu^2 + i\sin\theta\,\delta_\mu^3) \\ \overline{m}_\mu &= -\frac{r}{\sqrt{2}}\,(\delta_\mu^2 - i\sin\theta\,\delta_\mu^3) \end{aligned} \tag{2}$$

which satisfies $l_\mu n^\mu = -m_\mu \overline{m}^\mu = 1$, with all other inner products zero.

Using the Newman–Penrose (1962) formula, we can get from (1) and (2) the spin coefficients

$$\begin{aligned} &k = \pi = \epsilon = \lambda = \sigma = \nu = \tau = 0 \\ &\rho = -\frac{1}{r}, \qquad \alpha = -\frac{\cot\theta}{2\sqrt{2}r} = -\beta \\ &\mu = -\frac{1}{2r} + \frac{M}{r^2} - \frac{Q^2}{2r^3} + \frac{\Lambda r}{6} \\ &\gamma = \frac{M}{2r^2} - \frac{Q^2}{2r^3} - \frac{\Lambda r}{6} \end{aligned} \tag{3}$$

and the tetrad components of the Weyl tensor

$$\begin{aligned} &\Psi_0 = \Psi_1 = \Psi_3 = \Psi_4 = 0 \\ &\Psi_2 = -\frac{M}{r^3} + \frac{Q^2}{r^4} \end{aligned} \tag{4}$$

Equations (3) and (4) tell us that the Vaidya–Bonner–de Sitter metric is of Petrov type D.

3. THE GENERALIZED TEUKOLSKY MASTER EQUATION

Since the Vaidya–Bonner–de Sitter metric is a type D vacuum, the perturbation method of Teukolsky is applicable. Using the result of Teukolsky (1974), we obtain the linearized equations for the source-free case as

$$
\begin{aligned}
&[(D - 3\varepsilon + \bar{\varepsilon} - 4\rho - \bar{\rho})(\Delta - 4\gamma + \mu) \\
&\quad - (\delta + \bar{\pi} - \bar{\alpha} - 3\beta - 4\tau)(\bar{\delta} + \pi - 4\alpha) - 3\Psi_2]\, \Psi_0^B = 0 \\
&[(\Delta + 3\gamma - \bar{\gamma} + 4\mu + \bar{\mu})(D + 4\epsilon - \rho) \\
&\quad - (\bar{\delta} - \bar{\tau} + \bar{\beta} + 3\alpha + 4\pi)(\delta - \tau + 4\beta) - 3\Psi_2]\, \Psi_4^B = 0
\end{aligned} \tag{5}
$$

$$
\begin{aligned}
&[(D - \varepsilon + \bar{\varepsilon} - 2\rho - \bar{\rho})(\Delta + \mu - 2\gamma) \\
&\quad - (\delta - \beta - \bar{\alpha} - 2\tau + \bar{\pi})\, (\bar{\delta} + \pi - 2\alpha)]\Phi_0 = 0 \\
&[(\Delta + \gamma - \bar{\gamma} + 2\mu + \bar{\mu})(D - \rho + 2\varepsilon) \\
&\quad - (\bar{\delta} + \alpha + \bar{\beta} + 2\pi - \bar{\tau})(\delta - \tau + 2\beta)]\Phi_2 = 0
\end{aligned} \tag{6}
$$

where D, Δ, and δ are the directional derivatives, given by

$$
D = l^{\mu}\partial_{\mu}, \qquad \Delta = n^{\mu}\partial_{\mu}, \qquad \delta = m^{\mu}\partial_{\mu} \tag{7}
$$

Each pair of Eqs. (5) and (6) represents a graviton (spin $s = 2$) and electromagnetic ($s = 1$) perturbations. In each pair the first equation is for spin states $p = s$, while the other one is for $p = -s$.

For the Dirac field, the spinor base form of the field equation is given by (Page, 1976)

$$
\begin{aligned}
&(\nabla_{a\dot{b}} + ieA_{a\dot{b}})P^{a} + i\frac{\mu_0}{\sqrt{2}}\bar{Q}_{\dot{b}} = 0 \\
&(\nabla_{a\dot{b}} - ieA_{a\dot{b}})Q^{a} + i\frac{\mu_0}{\sqrt{2}}\bar{P}_{\dot{b}} = 0
\end{aligned} \tag{8}
$$

where μ_0 and e are the mass and charge of the particle, and P^a, Q^a, and $\nabla_{a\dot{b}}$ are, respectively, the 2-component spinors and the covariant spinor differentiation expressed with spinor base components. In terms of the spin coefficients, Eq. (8) can be written as four coupled equations

$$
(D + \varepsilon - \rho + ieA_{\mu}l^{\mu})F_1 + (\bar{\delta} + \pi - \alpha + ieA_{\mu}\bar{m}^{\mu})F_2 - i\frac{\mu_0}{\sqrt{2}}G_1 = 0
$$

$$
(\Delta + \mu - \gamma + ieA_{\mu}n^{\mu})F_2 + (\delta + \beta - \tau + ieA_{\mu}m^{\mu})F_1 - i\frac{\mu_0}{\sqrt{2}}G_2 = 0 \tag{9}
$$

$$(D + \bar{\varepsilon} - \bar{\rho} + ieA_\mu l^\mu)G_2 - (\delta + \bar{\pi} - \bar{\alpha} + ieA_\mu m^\mu)G_1 - i\frac{\mu_0}{\sqrt{2}}F_2 = 0$$

$$(\Delta + \bar{\mu} - \bar{\gamma} + ieA_\mu n^\mu)G_1 - (\bar{\delta} + \bar{\beta} - \bar{\tau} + ieA_\mu \bar{m}^\mu)G_2 - i\frac{\mu_0}{\sqrt{2}}F_1 = 0$$

where

$$F_1 = P^0, \qquad F_2 = P^1, \qquad G_1 = \bar{Q}^1, \qquad G_2 = -\bar{Q}^0 \tag{10}$$

A_μ is the electromagnetic potential. In the spherically symmetric space–time, it is given by

$$A_\mu = (Q/r, 0, 0, 0) \tag{11}$$

The Klein–Gordon equation is

$$\frac{1}{\sqrt{-g}}(\partial_\mu + ieA_\mu)[\sqrt{-g}g^{\mu\nu}(\partial_\nu + ieA_\nu)\Phi] + \mu_0^2\Phi = 0 \tag{12}$$

Using Eqs. (1)–(4), (7), and (11) and making the transformations

$$\Psi_0^B, \Psi_4^B, \Phi_0, \Phi_2, \Phi = r^{-(s+p+1)}\,{}_pR_l(v, r)\cdot{}_pY_l^m(\theta, \varphi) \tag{13}$$

and

$$\begin{aligned}
&[F_1, F_2, G_1, G_2]\\
&\quad = [r^{-1}\cdot{}_{-1/2}R_l(v, r)\cdot{}_{-1/2}Y_l^m(\theta, \varphi), r^{-2}\cdot{}_{+1/2}R_l(v, r)\cdot{}_{+1/2}Y_l^m(\theta, \varphi),\\
&\qquad r^{-2}\cdot{}_{+1/2}R_l(v, r)\cdot{}_{-1/2}Y_l^m(\theta, \varphi), r^{-1}\cdot{}_{-1/2}R_l(v, r)\cdot{}_{+1/2}Y_l^m(\theta, \varphi)]
\end{aligned} \tag{14}$$

we find that Eqs. (5), (6), (9), and (12) are separable into the forms

$$\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta} - \frac{1}{\sin^2\theta}\left(p^2\cos^2\theta - i2p\cos\theta\frac{\partial}{\partial\varphi} - \frac{\partial^2}{\partial\varphi^2}\right) - s + \lambda^2\right]{}_pY_l^m(\theta, \varphi) = 0 \tag{15}$$

$$\begin{aligned}
&\left\{A\frac{\partial^2}{\partial r^2} + 2\frac{\partial^2}{\partial v\partial r} + 2(s + p)\frac{i2p\mu_0}{\lambda - i2p\mu_0 r}\frac{\partial}{\partial v}\right.\\
&\quad + \left[(p + 1)A' + \left(\frac{i2p\mu_0}{\lambda - i2p\mu_0 r} - \frac{2p}{r}\right)A + i\frac{2eQ}{r}\right]\frac{\partial}{\partial r}\\
&\quad + (s + p)\frac{i2p\mu_0}{\lambda - i2p\mu_0 r}\left(pA' - \frac{A}{r}\right) + \frac{1}{6}(2p + 1)(p + 1)r\left(\frac{A}{r}\right)''
\end{aligned}$$

$$+\frac{1}{3}(2s-1)(s-1)\frac{1+(s-2)\Lambda r^2}{r^2}+(s+p)$$
$$\times\frac{i2p\mu_0}{\lambda-i2p\mu_0 r}\frac{i2eQ}{r}-\left(\mu_0^2+\frac{\lambda^2}{r^2}\right)\Bigg\}\,{}_pR_l=0 \tag{16}$$

where

$$A=1-\frac{2M(v)}{r}+\frac{Q^2(v)}{r^2}-\frac{1}{3}\Lambda r^2 \tag{17}$$

The prime denotes the derivative with respect to r. Equation (15) shows that ${}_pY_l^m(\theta,\varphi)$ is the spin-weighted spherical harmonic, and the separation constant λ satisfies

$$\lambda=\sqrt{(l+s)(l-s+1)} \tag{18}$$

Here l and m are integers satisfying the inequalities $l\geq s$ and $-l\leq m\leq l$.

Teukolsky (1973), using the Newman–Penrose formalism, succeeded in disentangling the perturbations of the Kerr metric, and wrote a master equation for the massless spin fields. Here we have derived the generalized master equation (16) governing not only the massless spin fields, but the massive scalar and Dirac fields as well. For the neutrino, we can set $\mu_0=0$ and $s=1/2$ in Eqs. (15) and (16).

4. A NEW QUANTUM EFFECT

In order to determine the tortoise-coordinate transformation in a nonstatic space–time, it is convenient to change variables to the set (Li and Zhao, 1995)

$$\tilde{r}=r-r_H,\qquad \tilde{v}=v \tag{19}$$

In these coordinates the metric (1) becomes

$$ds^2=\left[1-\frac{2M(v)}{r}+\frac{Q^2(v)}{r^2}-\frac{1}{3}\Lambda r^2-2\dot{r}_H\right]d\tilde{v}^2$$
$$-2\,d\tilde{v}\,d\tilde{r}-r^2(d\theta^2+\sin^2\theta\,d\varphi^2) \tag{20}$$

where r_H is the location of the event horizon satisfying the null-surface condition, namely

$$1-\frac{2M(v)}{r_H}+\frac{Q^2(v)}{r_H^2}-\frac{1}{3}\Lambda r_H^2-2\dot{r}_H=0 \tag{21}$$

and the dot denotes the derivative with respect to v. From the radial null condition, we have

$$d\tilde{v} = \frac{2(1-2\eta)}{1 - 2M/r + Q^2/r^2 - \frac{1}{3}\Lambda r^2 - 2\dot{r}_H} d\tilde{r} + 2\eta\, d\tilde{v}$$

$$= \frac{2(1-2\eta)}{A - 2\dot{r}_H} d\tilde{r} + 2\eta\, d\tilde{v} \tag{22}$$

Now, we define the differential form of the tortoise coordinate in the Vaidya–Bonner–de Sitter space–time as [using Eq. (22)]

$$dr_* = \frac{1-2\eta}{A - 2\dot{r}_H} d\tilde{r} + \eta\, d\tilde{v}, \qquad v_* = \tilde{v} \tag{23}$$

where η is an integrating factor, which satisfies

$$\eta \approx \dot{r}_H \qquad \text{(near the event horizon)} \tag{24}$$

Equation (23) can be integrated near the event horizon to be

$$r_* \sim \frac{1}{2\kappa} \ln (r - r_H) \tag{25}$$

where

$$\kappa = \frac{1}{1 - 2\dot{r}_H} \left(\frac{M}{r_H^2} - \frac{Q^2}{r_H^3} - \frac{1}{3}\Lambda r_H \right) \tag{26}$$

In order to solve the generalized Teukolsky-type master equation (16) near the event horizon, we have recourse to the coordinates (r_*, v_*), whereupon Eq. (16) becomes

$$\frac{\partial_p^2 R_l}{\partial r_*^2} + 2\frac{\partial_p^2 R_l}{\partial v_* \, \partial r_*} + (\Omega_0 + i2\omega_0)\frac{\partial_p R_l}{\partial r_*} = 0 \tag{27}$$

where

$$\Omega_0 = \frac{8(s+p-1)p^2\mu_0^2 r_H \dot{r}_H}{\lambda^2 + 4p^2\mu_0^2 r_H^2} - \frac{2p}{r_H}\left(1 - \frac{3M}{r_H} + \frac{2Q}{r_H^2}\right) \tag{28}$$

$$\omega_0 = \frac{eQ}{r_H} + \frac{2(1-s-p)p\mu_0\lambda\dot{r}_H}{\lambda^2 + 4p^2\,\mu_0^2 r_H^2} \tag{29}$$

The linearly independent solutions of (27) are

$$ {}_pR_l^{\text{in}} = \exp(-i\omega v_*) \tag{30}$$

$$ {}_pR_l^{\text{out}} = \exp(-i\omega v_*)\exp[2i(\omega - \omega_0)r_*]\exp(-\Omega_0 r_*) \tag{31}$$

Here ${}_pR_l^{\text{in}}$ is the ingoing wave and ${}_pR_l^{\text{out}}$ is the outgoing wave. According to Damour and Ruffini (1976) and Sannan (1988), the mass loss rate and the Hawking temperature of a Vaidya–Bonner–de Sitter black hole are given by

$$\frac{dM}{dv} = -\frac{1}{2\pi}\sum_{lmp}\int_0^\infty \frac{\omega\Gamma_{\omega lmp}}{\exp[(\omega - \omega_0)/T] \pm 1}\,d\omega \tag{32}$$

$$T = \frac{\kappa}{2\pi} = \frac{1}{2\pi(1 - 2\dot{r}_H)}\left(\frac{M}{r_H^2} - \frac{Q^2}{r_H^3} - \frac{1}{3}\Lambda r_H\right) \tag{33}$$

where $\Gamma_{\omega lmp}$ is the transmission coefficient in that mode, with which a particle can escape from the event horizon to infinity.

From Eq. (32), it is clear that the presence of ω_0 influences the radiative mechanism of the black hole. Equation (29) shows that ω_0 is the sum of two terms. The first term comes from the static electromagnetic field. We conjecture that the second term is due to the quantum ergosphere (York, 1983). There are two reasons to support this: (1) The second term is proportional to the physical quantity $\dot{r}_H$ describing the quantum ergosphere and (2) the second term depends on the spin state, which implies that the quantum ergosphere differs from the classical ergosphere of a rotating (Kerr) black hole. The results show that the nonstatic black hole should have some new quantum effects which are unknown as yet.

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